Metacognition Process Characteristics Of The Students In Solving Mathematics Problems

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Abstract: *The objective of this study was to analyse characteristic of students' metacognition process in solving mathematics problem. The process of the student metacognition analysed to describe its character is through the happening of awareness, evaluation, and regulation as metacognitin components. To describe the characteristic, researcher has determined the indicator of each component. And specially, the indicator is described into descriptor to explain the characteristic of metacognition process.* Based on the findings, the descriptos of every student during finishing calculus problem their characteristic of metacognition process can be known. *Researcher involves 23 students of mathematic education department who have finished Differential Calculus subject as the subject of the study. The data of the study are work result, think-aloud record, observation sheet, questionaire answer and interview transcript of each of the subject of the study. The result of the work is classified into high ability, midlle ability and low ability subjects. The data of the study obtained is analysed by using constant comparation method owned by Glaser and Strauss. Based on data analyse, it can be concluded that the characteristic of metacognition process is categoried into high ability, midlle ability, and low ability subject. With each indicator of awareness, evaluation, and regulayion appearing, the process happen completely and orderly, completely but unorderly, and uncompletely.*
Keywords: *awareness, evaluation, mathematics problem, metacognition, regulation.*

I. Introduction

Mathematics as a part of the instruction at school has direct and indirect objects. Gagne (Soedjadi, 2000; Hudojo, 2008) states that mathematical direct object relates to fact, concept, operation, and principle. The fact is a convention or agreement used to make mathematical discussion go smoothly, such as symbol, notation. The symbol "4" has been understood as figure "four". If being presented"4", people understand it is "four". On the other hand, if someone says the word "four", it is symbolized by "4". The concept is an abstract idea which can be used to classify a group of a certain objects. A concept in the field of mathematics is called mathematical concept. "Triangular" is a name of an abstract concept. With a concept, a group of objects can be classified as an example or not. The concept has a closely related to definition. Definition is an utterance which limits a concept. With it, someone can make illustration, picture, or symbol of defined concept. So that, it is clearer what is meant by a certain concept. Mathematical operation is procedures and as a process to find out a certain result. If the concept is an association, the possible operations are union, section, difference, or complement. Besides on association, it is known as addition operation, difference, multiplication, and division. The principle is the relation between various complex mathematical principle objects and consists of some facts. The right value principle is the principle has two or more concepts and states the relation between those concepts. For instance, the result of multiple of the figures p and q is zero if and if only p = 0 or q = 0.

Mathematical indirect object relates to the ability of logical thinking, solve problem, analytical thinking, positive thinking towards mathematics, careful, diligent, discipline and the other cases implicitly will be obtained if someone learns mathematics. Based on the opinion of Gagne above, it can conclude that one of the objects learned in mathematics is a concept. Mathematical concept orientation can be done through instruction. Djamarah (2008) states that someone who has possesses concept can do abstracting so that he/she can translate and call awareness and form mentally representation. During understanding the concept, someone needs the ability and strategy or certain way. Duffin & Simpson (2000) states that the ability possessed by someone during understanding the concept is expected to be able to re-express something communicated to the source of study, and finally, when the concept has been mastered, the given problems can answered.

Polya (1988) mentions four steps in solving problem, i.e. understand the problem, plan the treatment, execute the plan, and review the treatment done. The steps expressed by Polya in solving problem are activities which can be during the instruction and indicator in determine instructional result completeness. According to Bloom taxonomy completeness of learning result is classified into the field of cognitive, affective, and psychomotor. Especially for the field of cognitive, Anderson & Krathwohl (2001) improve into cognition process dimension and knowledge dimension. Cognition process consists of remembering, understanding,

applying, analysing, evaluating, and creating. But, knowledge consists of factual, conceptual, procedural and metacognition.

Metacognition comes from meta and cognition. Meta is a Greek means after or behind. Cognition is a process of obtaining knowledge (Zahmeister & Neyberg, 1982). During its process, metacognition can be related to the activities of problem solving, knowledge, cognition process and the strategy used during the instruction. The term of metacognition used for the first time by Flavell in 1976. According to him, metacognition consists of knowledge, experience and regulation which function as the important element and contribute the success of the problem solving.

The facts expressed by some high students during solving the problem that they often meet the failures. The failures were caused by the lack of the understanding of metacognition aspects, especially, which are related to the steps done to solve problem (Schoenfeld, 1992; Goos, 1995). As the improvement of the study, especially relates to the process of metacognition, it is found out that metacognition can assist one's thinking process more effectively and opened (Clarke, 2004). While Schoenfeld (1992) regards that the difficulty in soling problem is closely related to student's inability to observe and control the process of metacognition.

Some studies related to metacognition and problem solving has been done. Among of them are Desoete (2001), Lioe (2003), Wilson & Clarke (2004), Cromley (2005), Efklides (2006), Lesh (2007), Panauorra (2009), Kuzle (2011), Molenar (2011), Karan & Irizary (2011), Magiera & Zawojewski (2011), In'am (2012), Praba (2013), Zarimah &Tajudin (2013). The studies generally discuss the process metacognition in the subject of the study. However, it has not brought up the student's metacognition process characteristic in solving problem yet. Relating to the case, the researcher does the research to analyse and describes the mathematical student's metacognition process characteristic in solving Calculus problems. The student's metacognition process characteristic described based on the process of *awareness*, *evaluation*, and *regulation* which are the components of metacogntion.

II.   Theoritical Study
2.1. The Definition of Metacognition

Friedrichs & Hoyt (1976) mentions metacognition in term of *metamemory*. While Veenman (2012) describes metacognition as two main parts: the knowledge of metacognition and regulation or the observation of metacognition. The knowledge of metacognition is an interaction offering between someone's knowledge and the ability to do his/her assignments, characters of assignment, and the strategy which can be used to do the assignments. The regulation or metacognition observation is the activities relate to planning, monitoring, someone's evaluating, and the process of cognition to control the process. Metacognition as a process has four important aspects. According to Baker & Brown (1984) those aspects are self-controlling, planning, evaluating, and monitoring. Wellman (1985) states that metacognition is as form of cognition or two or more thinking process which include the control of the cognition activity. Therefore, metacognition can be said as thinking of someone's thinking of him/her-self or someone's knowledge of his/her cognition.

Besides having four aspects, according to Schoenfeld (1992), metacognition is as the process of someone's thinking process of what he/she has thought and as the interaction between the three important aspects, namely the knowledge of thinking process, self-controlling, and intuition. The interaction is very important for the knowledge of cognition process can assist and control the cases around us and select the strategies to improve our next cognition ability. According to Schoenfeld, the process of metacognition includes the ability to ask a question, and answer the question about a case, topic and problem, time allocation used to learn a certain topic, strategy, method and tactic used, level being learn by the student, fault done by the student, and revise the next plan.

Livingstone (1997) defines metacognition as *thinking about thinking*. In the other word, metacognition is someone thinking ability about what is thought. So that, metacognition object is thinking process happened to someone. Biryukov (2003) says that metacognition is someone thinking hypothesis of what has been thought before and includes knowledge, skill and experience. Knowledge is awareness of what is known, skill in the form of awareness of something done, and experience is awareness of cognition ability possessed.

Davidson & Sternberg (1998) state that metacognition has an important functions and contributes the success of problem solving that enable someone to identify and work strategically. Matlin (1998) states that metacognition is knowledge relates to awareness and cognition process. Wellman (1985) states that metacognition is as a form of cognition, or two or more thinking process including controlling of cognition activity. Therefore, metacognition can be said as someone thinking of self-thinking or someone cognition of self-cognition.

Tan (2003) states that metacognition is thoughtfulness refers to think of self-thinking, self-checking, and information process and how to process information effectively. Lioe (2003) states that metacognition is someone awareness about cognition process and selveness to reach a certain goal. Metacognition appears in problem solving whose components are attitude, skill, concept, process, and metacognition.

Peirce (2003) defines metacognition generally and specifically. Generally, metacognition is thinking of thinking. While specifically, Peirce quotes the definition of metacognition by Taylor, which states that metacognition is the appreciation of what has known which relates to the ability to make a right conclusion about how to apply someone's strategy knowledge of certain situation, and to do it efficiently and accurately. Taccasu (2008) describes metacognition as a part of planning, monitoring and evaluating the instructional process, thinking what we have known or unknown and control how the instruction include both awareness and aware control someone's learn so that it is effective. Mokos & Kafoussi (2013) states that metacognition is someone ability to observe and control him/her-self toward the case known. During the mathematical instruction, the important thing is the process of the study of student's metacognition in doing the problem more focussed to the problem solving field relates to mathematics.

Based on some definition stated above, we can identify the main meanings of metacognition: (1) is soul ability in cognition group, (2) is ability to aware, know, cognition process happened in oneself, is ability to direct cognition process in oneself, (3) is an ability of how the instruction done including the process of planning, observing, and evaluation, (4) is an activity of high level thinking activity for its activity can control thinking process happened in oneself in the present time, and (5) relates to student's thinking process to find out suitable strategy in solving problem, (6) metacognition skill is very important in solving mathematical problem, so the skill needs to improve. To improve metacognition skill, student needs awareness in his/her thinking steps. Student awareness in thinking is needed to finish a problem.

2.2 The Components of Metacognition

Magiera & Zawojewski (2011) find that metacognition activity happened during giving assignment in the class. Metacognition happening in the students has three components, e.g. *awareness, evaluation*, and *regulation*. During the process of metacognition, it can be seen the appearing activities in every components of metacognition called as types of metacognition activities. The types of awareness consist of what the students know, what the students need to solve the problem, what the students must do, where the students solve the problem. The types of *evaluation* consist of result evaluation, students' difficulty problem of evaluation, progress, ability or understanding. The types of *regulation* consist of planning strategy, selecting strategy of problem solving, formulating the goal.

Awareness, according to Wilson & Clarke (2002, 2004), relates to someone's awareness in the process of learning or in the process of solving problem, the content of specific knowledge owned, and someone's knowledge in learning or strategy in solving problem. It also includes someone's knowledge about what is needed to do, and what has done, and what can be done in a certain learning or situation in solving the problem. *Evaluation* refers to evaluation made by someone about thinking process, ability and limitation, such as working in a certain situation or as a self-complication. For example, someone can make evaluation about thinking effectiveness done or strategy chosen. *Regulation* in metacognition happens when someone uses his/her skill of metacognition to direct knowledge and thought and refers to individual knowledge in the form of strategy, such as how and why using certain strategies, as well as skill, such as planning, self-correction, decide the goal to optimal the usage of their own cognition source.

Metacognition components stated by Wilson & Clarke (2002, 2004) and Magiera & Zawojeski (2011) have indicators as variable and measurement. However, how the process of changes among the components of metacognition has not analysed deeper yet. Sriraman (2003) has considered students about the relationship between their knowledge and what is needed in problem situation given, as Stillman & Gabraith (1998). *Evaluation* has been described and studied concerning the students explicit reflection during the process of solving problem, the function of evaluation in determining the strategy in solving problem. Make a decision in systematic evaluation, alternative plan, and strategy in solving problem (Lester, 1980; Lester, Garofalo, & Kroll, 1989). *Regulation* has been clarified in the form of student flexibility in choosing a solution plan, choosing strategy, and plan implementation improved by Lester (1989), Zan (2000).

III. The Method Of The Study

The study done is qualitative descriptive whose subject is 23 mathematical students who have got Differential Calculus as the subject of the study. The subject of the study consists of 6 high ability students, 9 middle ability students, and 8 low ability students. The method of the study is arranged in steps (1) the researcher gives students calculus problem in the form of function downward application to determine maximum and minimum value. The problems have been validated by 2 experts in mathematics and mathematics education. During doing the calculus problems, the subject do *think-aloud* and being recorded. The problems given to the subject of the study is enclosed in the article. (2) Researchers do the observation to the students during the process of *think-aloud* using observation sheet to make known the appearance of indicator and descriptor of *awareness, evaluation,* and *regulation.*(3) Researcher corrects the result of student's work based on the answer-key made before. Based on it, result of the work of the subjects of the study are classified into high,

middle, and low ability students.(4) Giving questionnaire. It is given after the students finish doing calculus problem. It consists of 14 items to measure the process of metacognition happened to the students. It is made as the improvement of research done by Biryukov (2001), Azsoy & Ataman (2009), Meriam & Idrus (2010), Panaoura (2010), Sengul & Katransi (2013). (5) Doing the interview. It is based on the result of students' works, *think-aloud* record, and questionnaire answers. It is done to make deeper known about the characteristic of the process of *awareness, evaluation*, and *regulation*. It is done after students carry out *think-aloud*. Interview protocol is arranged by improving the indicators of *awareness, evaluation*, and *regulation*. (6) Doing transcription ofthe record of *think-aloud* and intervie**w**. The transcription is done for obtaining the data of each of the subject of the study of the characteristic of metacognition process during finishing calculus problem relating to students ability.(7) Data Reducing. Reducing the data is done by making abstract in the form of summary of data core, process, and statements done by the subject of the study in finishing calculus problems as well as arranging the data in parts which will be categorised by *coding*. (8) Data analysing. Analysing metacognition process of each of the subjects of the study done through each indicators *awareness, evaluation*, and *regulation*. (9) Data validation. Data validation is done using triangulation technique and checking classmates through discussion. Triangulation used is source one, that is comparing the result of observing during the subject finish calculus problems, observation sheet, the result of students works, *think-aloud*, questionnaire answers, and interview result.

IV. The Result Of Study And Discussion

The data of the study is in the form of the result of works, the record of *think-aloud*, the observation sheet, answers of metacognition questionnaire and the interview transcript is studied and analysed qualitatively. The theory design built by the researcher is analysing the characteristic of the student metacognition process in finishing calculus problems through indicators of *awareness*, *evaluation*, and *regulation.* Based on the theory design, the result of analyse of the study is grouped into category of characteristic of student metacognition process of high, middle and low ability.

The result of the study is from 23 students of Mathematics Education Department who had Differential Calculus. They were given calculus problems and during finishing it appeared the components of *awareness, evaluation*, and *regulation.* Besides the metacognition appearance components completely, the subject of the study were given similar petterns so the next analyse is classify the subjects into high, middle, and low ability. Based on it, 23 subjects of the study are classified into 6 for high ability, 9 for middle ability, and 8 for low ability.

Based on the result of data analyse using the method of constant comparation by Glaser and Strauss and data validation using triangulation in which each of the subject of the study in each group has relatively same characteristic. Sothat, data explanation was done to 6 subject of the study. They are S-1 and S-2 of high ability group, S-3 and S-4 of middle ability group, and S-5 and S-6 of low ability group. The next step is studying the characteristic of metacognition process of those three groups of subject of the study through their component indicators of *awareness*, *evaluation*, and *regulation*.

The component of *awareness* has 5 indicators. They are the subject of the study rethink of what known about calculus problem given (A1), rethink of the calculus questions and match with similar problem which obtained and finished before. (A2), rethink of cases which have not been solved before (A3), rethink of the next step to do to finish calculur problem given (A4), and rethink of steps done when answering calculus problem (A5).

The component of *evaluation* has 5 indicators. They are the subject of the study rethink the way used to finish calculus problem given, (E1), rethink of the order and steps to do when finishing calculus problem given (E2), recheck the result of answers of calculus problem finished (E4), and rethink failure done in answering calculus problem given using previous way (E5).

The component of *regulation* has 4 indicators. Rethink and make a plan to finish calculus problem given soon (R1), rethink the different way used in answering calculus problem given (R2), rethink of what will be done next after finishing calculus problem (R3), and rethink how to change the way in finishing calculus problem given (R4). The process of happening of metacognition of the subject of the study and their characteristics can be explained as follows.

4.1 The Process of High Ability Student's Metacognition

The subject of the study S-1 and S-2 have relatively same metacognition process characteristic. During metacognition process, S-1 and S-2 show the activity which describes the characteristic of the component indicators of *awareness*, *evaluation*, and *regulation*. When finishing calculus problems, the indicators appear in order A1, A2, A3, A4, A5, E1, E2, E3, E4, E5, R1, R2, R3, and R4.
The explanation of each descriptor and the appearance of indicators S-1 and S-2 can be seen in figure 4.1.
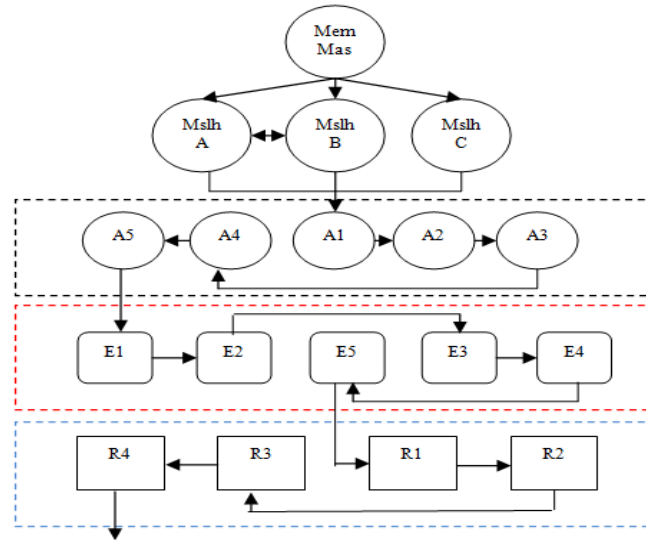
Figure 4.1: Metacogniton Process S-1 and S-2

4.2. The Process of Middle Ability Student's Metacognition

The middle ability subject of the study, S-3 and S-4 carried out metacognition process with complete indicators but disorder. When finishing the problems, the indicator in S-3 appeared in order A1, A2, A3, A5, A4, E2, E1, E3, E4, E5, R1, R3, R2, and R4. The indicator in S-4 appeared in order A1, A2, A3, A4, A5, E1,E2, E3, E4, E5, R1, R2, R4, R3.

The order of appearing indicator in S-3 can be seen in figure 4.2 and the order of appearing indicator in S-4 can be seen in figure 4.3
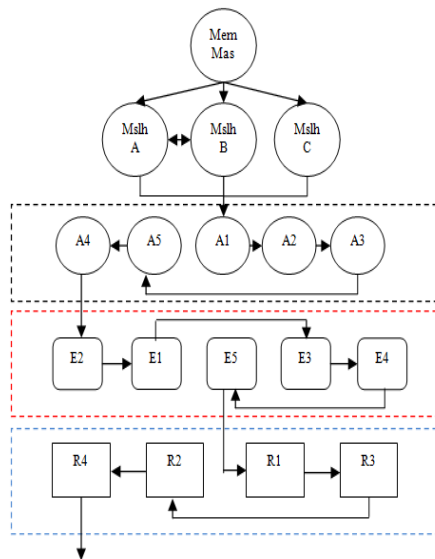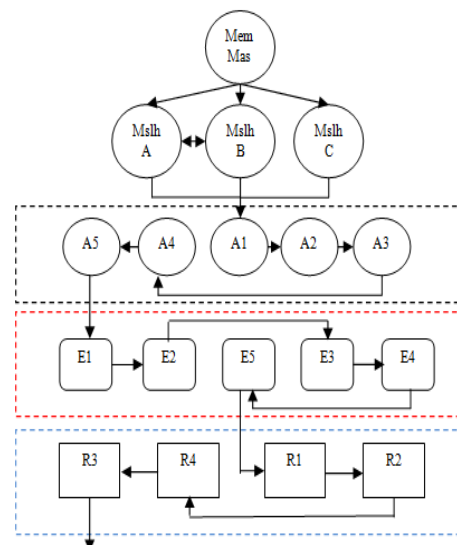

Figure 4.2 Metacognition Process S-3   **Figure 4.3 Metacognition Process S-4**

The low ability subjects of the study are S-5 and S-6. The change of metacognition process S-5 and S-6 show the activity which describes the characteristic of indicators *awareness, evaluation*, and *regulation*. When finishing calculus problem, the indicator S-5 appears in order A1, A3, A2, A4, A5, E1, E2, E3, E4, R1, R3, R2, R4. The indicator S-6 appears in order A1, A2, A3, A4, E1, E2, E3, E4, E5, R1, R2, R4, R3.

The order of indicator in S-5 can be seen in figure 4.4 and the order of indicators in S-6 can be seen in figure 4.5
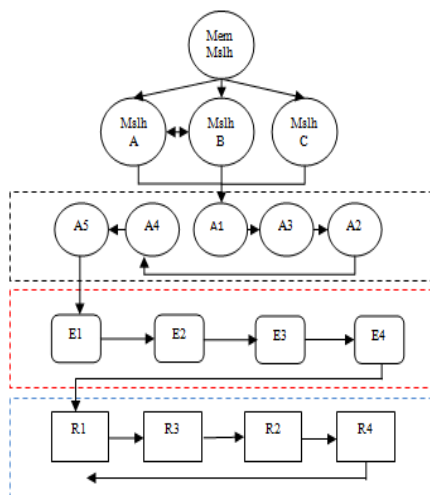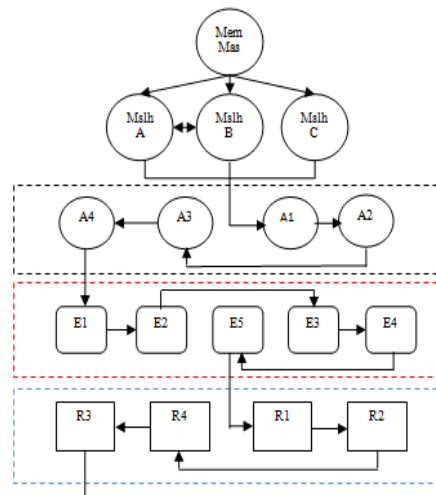
Figure 4.4 Metacognition Process S-5   **Figure 4.5 Metacognition Process S-6**

The finding of the result of the study above can be related to the previous theory as the basic of the study done. If being related to Wilson dan Clarke's opinions (2002, 2004), each of the characteristic of each components are activities relating to someone's ability to be able to think what have been done when finishing the problem.The problem of the study is how the subject of the study can find out a believed step as the answer of calculus problem given. The believeness of the subject of the study is based on the process of evaluation and regulation done and as part of abilities to observe her/him-self toward a case known, self-control can be done when finishing problem (Mokos & Kafoussi, 2013). Concerning to the indicator during the process of metacognition, it can be seen that activity as types of metacognition activity.The types of awareness activities are what are known by student, what is needed by the student to solve the problem, what must be done by the student, where the student is when finishing the problem. The types of evaluation activities are evaluation of the result, evaluation of the problem, student's difficulty, the evaluation of the progress, ability or understanding. The types of regulation are the strategy in planning, choosing strategy in solving problem, formulating the goal (Magiera & Zawojewski, 2011).

V.   Conclusion

Based on the data analyse, it can be concluded that the high ability student metacognition process characteristic in solving calculus problem is complete and in order, the middle abilty student is complete but disorder, and the low ability student is uncomplete.

The characteristic of complete and in order metacognition process is in S-1 and S-2, the characteristic of complete and disorder metacognition process is in S-3 and S-4,the characteristic of uncomplete metacognition process is in S-5 and S-6 can be explained as follows: The componenet of *awareness* with A1 brings up 6 characteristics, A2 brings up 7 characteristics, A3 brings up 5 characteristics, A4 brings up 8 characteristics A5 brings up 4 characteristics. The component of *evaluation* with the indicator E1 brings up 4 characteristics, E2 brings up 5 characteristics, E3 brings up 6 characteristics, E4 brings up 4 characteristics, and E5 brings up 4 characteristics. The component of *regulation* with the indicator R1 brings up 5 characteristics, R2 brings up 4 characteristics, R3 brings up 4 characteristics, and R4 brings up 6 characteristics

The whole characteristics which appearing at each of the subject of he study can be explained in the following table.

TABLE 1: The Finding of Indicator and Characteristic of Metacognition Process

Component of Awareness		
Title	**Indicator**	**Characteristic**

A1	Rethink of what is known of calculus problem given.	The subject of the study:
		        Read the given problem repeatedly and give marks at the words which are regarded as the key-words.
		        Note the important cases of problems by underlining the words which are regarded as the key-words.
		        Check the figure in the problem and represent the length and width of the figure as variable x and y.
		        Read the table of problem and note it as the known case and differenciate the volume for the field as part of problem known.
		        Make an important note and conclude known cases as the requirement to determine the

Title	Indicator	Characteristic
		way of solving the problem given. Make a correlationship between the cases known in the problem and each questions which will be answeredat the problem given
A2	Rethink the questions of calculus problems and similar problems which are finished before.	The subject of the study: Read each question repeatedly dan relates them to the previous questions of problems as known cases. Give marks at the words which are regarded as the key-words and each questions and conclude cases asked in problems. Make relationship question (a) in the problem relating to the area of rectangular. The relationship between area (L) and length and width is $L = xy$ or $L = 5xy$ depends on the assumption done for the length and width which are assumed before. Make relationship between the length of available barbed wire (s) on the problem and the fanced stable in the form of equation $s = x + 6y$ or $s = 5x + 6y.$ Since the length of barbed wire is 240 meters, there are two equations can be written: $240 = x + 6y$ or $240 = 5x + 6y.$ State the area of rectangular at the problem known as one function consisting of long and wide variable known as area function variable. Change the function of rectangular area on the problem given as one function variable by determining the maximum requirement of a field area. Conclude the cases asked in the problem based the note made.
A3	Rethink of cases which have not been finished in the last time yet when finishing calculus problem given.	The subject of the study: Make the relationship between the length of barbed wire and wired stable part as equation $240 = x + 6y$ or $240 = 4x + 6y.$ State the area of stable in form of one changer function as $L = xy$ into $L = x(240 - 6x).$ State the area of stable in the form of one changes function as $L = 5xy$ into $L = 5(240 - 6y/5)y.$ Do the substitution of area function in one changer and determine downward (*derivative*) of the function. Write downward of one changer function and determine the requirement of maximum and minimum value of a function as $dL/dx = 0$ or $dL/dy = 0$
A4	Rethink the next step to do to finish calculus problem given.	The Subject of the study : Rewatch cases known in the problem. Chose the way used to do the problem given based on the knowledge got before. Determine variable value decided after downwarding the area function stated in one changer function. Do variable substitution which has got into the previous function, i.e. $240 = x + 6y$ or $240 = 5x + 6y$ Find out the whole stable area and each stable as the answer of the problem given. Reread table at the problem given to check the truth of the answers got. Write the area comparison of each stable and the amount of goats. Write the fee of stable building material and the fee of weekly operational as stated in table of problem given.
A5	Rethink the explanation of the answers of calculus problem given.	The subject of the study: Reread the answers of the problem to make known the order and systimatical answers of problem as being asked and required at the questions (a), (b), and (c). Try to use another way to answer the problem which has not done before. Use the new way repeatedly and check the cases known in the problem. Compare the answers explanation between one way and the other which has been done before.
	Component of Evaluation	
Title	**Indicator**	**Characteristic**
E1	Rethink the way used to finish calculus problem given,	The subject of the study: Compare the result of each way done in answering the questions at the problem given. Mark the important things at the way sand steps used to solve the problem. Note the important cases at the difference between the used to solve the problem given. Make the relationship between cases known and the ways used to finish problem.
E2	Rethink of the order of steps to do at the time finishing calculus problem given.	The subject of the study: Recheck the relationship between the cases known and cases asked in the problem. Check the explanation of the answers written based on in the ways done. Recheck the relationship writing beween the length of wire avaible at the problem given and part of the fence which is fenced. Read the table and make an orderof the amount of goats in each stable with the area of each stable. Mark the important cases on the steps of answering problem done.

E3	Check the answers of calculus problems finished.	The subject of the study:
		    Check maximum and minimum requirement of one changer function based on the area formula written before.
		    Recount the length and width of the whole stable and each of the stable at the problem given.
		    Recount the comparation between the area of the stable and the amount of goats in each stable.
		    Recount the amount of the building stable material.
		    Recount the operational fee of the whole stables weekly.
		    Conclude question answers thoroughtly.

E4	Rethink of the truth of the calculus answers available.	The subject of the study:
		    Reread the question answers (a), (b), and (c) to make known whether the answers written on the answer sheet are right or not.
		    Make a relationship between answers obatained and each problem given.
		    Correct the length and width of the stable obtained with the length of the wire available.
		    Recount problem (b) and (c) by rewatch table 1 on the calculus problem.

E5	Rethink the failure done in answering calculus problem given by previous way.	The subject of the study:
		    Reread the answers of each of the question to know the truth value.
		    Recheck the answers about the requirement fillfulled maximum and minimum value of the area of whole stable in the question (a)
		    Recheck question answers (b) about comparation between the amount of stable and the area of the stable as the requirement at figure (1) and table (1) on calculus problem.
		    Recheck the answers (c) about total opearational fee each week and stable bulding fee as being required on table (1) on the calculus problem.

Component of Regulation		
Title	**Indicator**	**Characteristic**

R1	Rethink of making a plan to finish calculus problem given soon.	The subject of the study:
		
		    Recheck the problem answer repeatedly before making conclusion.
		
		    Decide the proper and easy way to answer problem questions given.
		
		    Order the answers of each questions based on the requirement known before.
		
		    Mark the failure in counting the problem answers given.
		
		    Reread table and figure in the problem given to determine the easy and proper way used to answer the problem given.

R2	Rethink different way used to answer calculus problem given.	subject of the study:
		
		    Check the way used to answer the problem question.
		
		    Differenciate the way used to finish calculus in problem given.
		
		    Order question answers of each question in the problem given.
		
		    Use the easy way in answering questions to explain the answers of each problem.

R3	Rethink of what will do to start answering calculus problem.	subject of the study:
		
		    Reread the known cases in the problem given.
		
		    Check the contability of data at the table and figure with the aswers given in the problem given.
		
		    Determine and decide the easy way wich can be used to answer the problem given.
		
		    Determine the answer problem question by making relationship between the known cases and asked.

R4	Rethink how to change the way in finishing calculus problem given.	The subject of the study:
		    Recheck the answers of problem questions given and compare the questions to rectangular figure known.
		    Mark the important casess of the answers known.
		    Make a conclusion at the answers using the way done.
		    Check the answer difficulty level done.
		    Recheck the answers of the problems given at answer sheet of the subject of the study.
		    Conclude each question answer (a), (b), and (c) of questions given.

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Appendix A
Mathematics Problems
Mr.Syarif has 100 goats. They will be entered into indentical rectangular stable as Figure 1 and the amount of each stable as written in table 1.
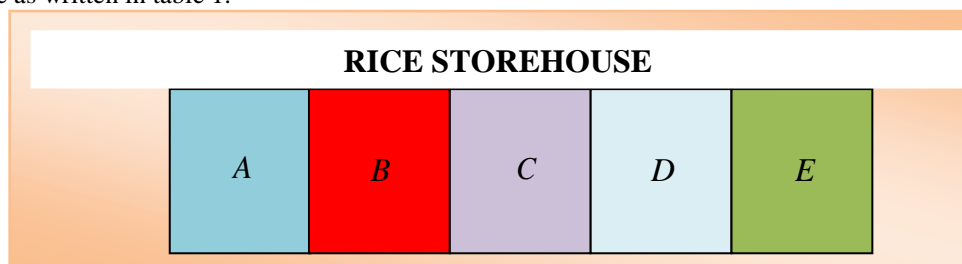
RICE STOREHOUSE

| A | B | C | D | E |

Figure 1 Sketch of Mr.Syarif's Stable

Mr.Syarif's stable lies behind rice storehouse owned by a businessman. It will be fenced with barbed wire to guard his goats.
Because being behind a rice storehouse, the part which is bordered with rice storehouse doesn't need to fence. Mr.Syarif prepares 240 meters barbed wire to fence his stable.
Entering goats into the stable, Mr.Syarif makes a note as follows:

Table 1: Comparation and Content of Mr.Syarif Stable

	Stable A	Stable B	Stable C	Stable D	Stable E
Amount of goats	20	24	25	16	15
Fee of stable material	Rp.500.000,-	Rp.500.000,-	Rp.500.000,-	Rp.500.000,-	Rp.500.000,-
Fee of goat daily operational of each stable	Rp.150.000,-	Rp.160.000,-	Rp.165.000,-	Rp.145.000,-	Rp.145.000,-

Do the following problems based on the text above!
a.   Measure Mr.Syarif's stable to make all stables as wide as possible!
b.   State the comparison between the amount of goat in each stable and the width of each of stable!
c.   Count the fee of weekly operational needed by Mr.Syarif and the fee of the whole stable!
(Adapted and modified from the Book of Calculus volume I by Edwin J. Purcell, Dale Verberg, dan Steven E, Rigdon in 2004 page 172).

Apendix B

The Questionnaire
Put a cross (X) in the column alternatives according to your choice

No	Statement	Alternative		
		Yes	No	Unsure
1.	I read the problem more than once			
2.	I checked that I understood what the problem was asking me			
3.	I assessed how much time I need to solve this problem			
4.	I represented the problem schematically			
5.	I tried to remember whether I had worked on the problem like this before			
6.	I've built a strategy for solving the problem			
7.	I did not know how to begin			
8.	During solving the problem I encountered a difficulty (if "Yes", describe the character of the difficulty)			
9.	During solving the problem I found a mistake and corrected it (if "Yes", describe the mistake)			
10.	I thought about how I was going			
11.	I tried different approaches for solving the problem			
12.	I asked myself whether my answer made sense			
13.	I checked my calculations to make sure they were correct			
14.	I thought whether there was something in the information that was given in the problem that needed special attention (if "Yes", describe it)			